

Mathematics and Language, or Knowledge as Semiotic Process

5.8. 2019

Michael Otte

I.

The world can be validly construed either as a universe of facts and a forum for interpretation or it can be seen as a place of things. Wittgenstein maintained that,

“the world is the totality of facts, not of things. The world is determined by the facts. The facts in logical space are the world” (*Tractatus*, 1.1).

So, the world is just a linguistic construction. We must consider the so-called “linguistic turn of analytical philosophy” as essential and according to Michael Dummett,

“there can be no doubt that, although to a great extent Frege and Russell prepared the ground, the crucial step was taken by Wittgenstein in the *Tractatus Logico-philosophicus* of 1922 (Dummett, M. *Origins of Analytical Philosophy*, Bloomsbury Revelations, p.121).

Wittgenstein concluded that if there were no language there would be no logic. And this would mean that there were no necessity, since all necessity is logical or linguistic necessity.

Wittgenstein’s friend Frank Ramsey pointed out to him, however, that the impossibility of a particle being in two places at the same time expresses a feature of the world, rather than of language.

If one wants to characterize knowledge or the world as a **semiotic process**, the restriction to the linguistic sign is not sufficient. Mathematics is no language. It is exactly this indispensability of **indices** that Frank Ramsey was pointing out to Wittgenstein.

Now Bolzano put his whole Doctrine of Science on exactly the very same foundations as Wittgenstein.

And Bolzano had his Ramsey too, namely Kant!

It is common knowledge that what Bolzano says or proves or deduces or claims should be always considered in the *context of linguistic representation*. This has often been criticized or misunderstood. For example Palagyi wrote in 1902:

"The extreme pursuit of liberation of thought from all its subjective conditions has in Bolzano the effect that we in all our thoughts become to an even greater extent slaves of our verbal expressions" (Palagyi, M. 1902, *Kant und Bolzano*, Verlag Niemeyer, Halle, p.76).

And Hugo Bergmann was saying that Bolzanos definition of analyticity signifies a complete „surrender to the arbitrariness of linguistic maneuvers” (Bergmann, H. 1909, *Das Philosophische Werk Bernard Bolzanos*, Halle a.S.: Niemeyer, p.76).

In fact, Bolzano grounds his entire *Doctrine of Science* on a single sentence. Namely on the sentence, *There are true sentences*.

This axiom cannot be refuted, because the sentence, “*There are no true sentences*” is itself a sentence and as such it is either true or false. Assuming that it is true, leads to contradiction.

It seems interesting indeed to notice that Bolzano explicitly claims historical *originality* for this kind of grounding. Bolzano was after all a self-confident but modest man. But he says:

“I really do not remember that the sentence *Nothing is true*, leads to a contradiction *only* if one assumes that it is itself a sentence has already been realized by anybody else” (WL, §33, I., p.151),.

For example, the classical skepticism in the sense of *Sextus Empiricus*, doubting or negating that there are truths in themselves, already implies - as Bolzano observes - the "ability to recognize truths" (WL, §33, I., p.148).

The difference between *Sextus Empiricus* and Bolzano lies exactly in the means of thinking and communication. And this seems very important.

Infact reflecting about knowledge from the point of view of possible means of cognitive activity seems to be a fundamental choice. Paul Valery once said

“Mastery presupposes that one has the habit of thinking and combining directly from the means of activity, of imagining a work only within the limits of the means at hand, and never approaching a work from a topic or an imagined effect that is not linked to the means” (Valery, Leonardo, p. 40).

In artistic drawing what we achieve is a line, and the line does all the work, and if it fails, no philosophical commentary will rescue or repair a bad work of art. In literature or philosophy, it is the word or the sentence, in mathematics the formula or the diagram, which carry the entire weight, etc. etc.

And the great music director David Barenboim means the same when observing that “music does not work with spiritual means, although it aims at the spiritual possibilities of man. Music works with sound. Sound is not spiritual, it is a very simple physical means. Sound is related to silence” (interview in: *Der Spiegel*, 12/2014, our translation).

David Hilbert meant the very same, when he said in 1922 in Hamburg:

“*Am Anfang ist das Zeichen*” (*Hilbertiana*, WBG Darmstadt 1964. P.18).

And so did Wittgenstein ends his Tractatus:

“7. Wovon man nicht sprechen kann, darüber muss man schweigen.-

Whereof one cannot speak, thereof one must be silent”.

If one does not take the necessity of a semiotic representation seriously, one might easily identify the arguments of Sextus Empiricus and Bolzano! And that is what Bolzano denied.

Bolzano constitutes a kind of linguistic ontology by hypostatizing linguistic sense and thereby conceiving of a “third world”, beyond the world of physical objects or physical states and the world of states of consciousness. Popper, who had coined this term, “third world”, had acknowledged inspiration from Bolzano in his search for an “objective epistemology”, that is, of an epistemology without a human subject.

II.

Classical knowledge before the *Scientific Revolution* of the 17th century was completely determined by its object. Thinking meant thinking of Being itself, that is, thought was determined by its object.

Cassirer writes:

“The Aristotelian logic, in its general principles, is a true expression and mirror of the Aristotelian metaphysics. Only in connection with the belief upon which the latter rests, can it be understood in its peculiar motives. ... In the further development of logic, however, its connections with the Aristotelian ontology in its special form begin to loosen..... Every attempt to transform logic.... is comprised in criticism of the general doctrine of the construction of concepts” (Cassirer, E. 1953, *Substance and Function*, N.Y. Dover, p.4, the German original publication is from 1910: *Substanzbegriff und Funktionsbegriff*, Berlin).

At a certain period in history it happened that that concepts. became understood as **functions** of human activity, rather than **abstractions** or abstract images of things.

Words and things parted ways.

Michel Foucault expresses these changes as the transition from “*the Age of Interpretation*” to the “*Age of Representation*”. He writes:

“At the beginning of the 17th century writing has ceased to be the prose of the world, resemblances and signs have dissolved their former alliance, similitudes have become deceptive. And just as interpretation in the sixteenth century ... was essentially a knowledge based upon similitude, so the ordering of things by means of signs constitutes all empirical forms of knowledge as knowledge based upon identity and difference” (Foucault, M., 1973. *The Order of Things*, Vintage Books, N.Y, p.47 -51 and pp.56-57).

The separation of sense and reference promoted a search for more secure methods.

“Since the late 16th century, more and more authors opt for the certainty of the method and the mathematical method therefore gains in importance, because it is the safest” (Schüling, 1969, *Die Geschichte der axiomatischen Methode im 16. und beginnenden 17. Jahrhundert*, Olms-Verlag, Hildesheim, p. 76).

Benjamin Nelson has pointed out that the main impact of the *Scientific Revolution* of the 16th/17th century came from a change in the habits of thought and, in particular, from a campaign for individual cognitive certainty.

It was the central problem of Descartes and of Leibniz.

Therefore *Classical Rationalism* required **God as a guarantor** of the connection between the signs and the objects. And proofs of God’s existence became important. Leibniz tried, like Descartes and many others, to frame such a proof:

“An argument for the existence of God, celebrated among the Scholastics long ago and revived by Descartes, once led me to consider this point more distinctly. The argument goes: whatever follows from the idea or definition of anything can be predicated of that thing. Since the most perfect being includes all perfections, among which is existence, existence follows from the idea of God. Therefore, the existence can be predicated of God.

But we must realize that from this argument we can conclude only that God is possible, from this argument we cannot safely use definitions for drawing conclusions unless we know first that they are real definitions, that is, that they include no contradictions” (Leibniz, *Philosophical Essays*, Indianapolis, Hackett Publ. Co. (Ariew/Garber (eds.), 1989, p. 25).

Kant objected to Leibniz’ reasoning, because, in his view, existence is not a property of anything, to say of some x that it exists adds nothing to the concept of x . Why not? Because the concept is not just a collection of some characteristics of the related object, contrary to the view commonly held since Aristotle. The abstraction theory of concepts is insufficient, because this theory underrates the autonomy of sense.

The principle of consistency only applies, according to Kant, if there is an object given. The statement that “a triangle has three angles does not enounce that three angles necessary exist, but upon the condition that a triangle exists, three angles must necessarily exist in it” (Kant, B 622).

In contrast, Leibniz obviously believed that the existence of an object is a consequence of the consistency of its concept, because the world was just a kind of idea in God’s mind..

Leibniz had in fact in his exchange of letters with Bishop Clarke, used the very same example as Ramsey. But had put it upside down. Leibniz's argument was directed against the Newtonian idea of absolute space and it can be rephrased as follows:

Suppose that space is absolute. Since every region of space would be indiscernible from any other and spatial relations would be construed as **extrinsic**, it would be possible for two substances to be indiscernible, yet distinct in virtue of being in different locations. But this is absurd, Leibniz argues, because it violates the principle of the *Identity of Indiscernibles*.

Kant rejected Leibniz' principle because a mere conceptual descriptions cannot capture all there is to spatial relations. The two gloves example demonstrates this. If you take a right-handed glove and a left-handed glove, all the relations between the various parts of the two gloves are exactly the same, but they represent different *orientations* in space (Kant, Immanuel (1768): *Von dem ersten Grunde des Unterschiedes der Gegenden im Raum*, Werke vol.I., Frankfurt Suhrkamp Verlag, p.993-1002).

From such or similar observations results Kant's assertion that all human knowledge "springs from two fundamental sources of the mind; the first is the capacity of receiving representations ..., the second is the power of knowing an object through these representations Through the first an object *is given* to us, through the second the object is *thought* in relation to that [given] representation. ... Intuition and concepts constitute, therefore, the elements of all our knowledge" (*Critique of Pure Reason* 1787, p. 74f.).

III.

Both Kant as well as Bolzano understood that the extensions and intensions of concepts are relatively independent of one another and that the transition from the possible to the factual cannot be accomplished by means of logic or language and pure thinking.

Bolzano also agreed with Kant's rejection of the *pre-established harmony* between being and knowledge. "It had exactly been Bolzano, who ... had completely anti-platonically distinguished between the structure of being and the structure of cognition (Denkstruktur)" (Neemann, U. 1972, *Bernard Bolzanos Lehre von Anschauung und Begriff*, Schöningh Paderborn, p. 81).

Bolzano also recognizes and values Kant's insistence on the **analytic/synthetic** distinction as important and he drew a sharp a distinction between concept and object, like Kant. And on this distinction the other one between analytic and synthetic propositions is crafted, because it has made both Kant as well as Bolzano, aware of the errors of the traditional notion

of a concept as something established by abstraction, wherefrom results the law of inverse relation between content and extension of concepts.

Bolzano, refusing this law of inverse relationship, writes:

„If I am so fortunate as to have avoided a mistake here which remained unnoticed by others, I will openly acknowledge what I have to thank for it, namely it is only the distinction Kant made between analytic and synthetic judgments, which could not be if all of the properties of an object had to be components of its representation (Vorstellung)“ (WL, §120, I., p.571).

A (true) proposition is obviously synthetic, if its predicate contains a characteristic of the object, which is not already part of the presentation of the subject of that proposition.

Bolzano's definition of analytical propositions is as follows:

“If there is a single representation (eine einzige Vorstellung) in a proposition which can be arbitrarily varied without disturbing its truth or falsity ... then this character of the proposition is sufficiently remarkable to distinguish it from all others. I permit myself thence to call propositions of this kind, borrowing an expression from Kant, *analytic*, all others, however, *synthetic* propositions” (Bolzano, WL §148, p.83).

A proposition is either true or false; and this permanently so. That certain propositions, like “this flower smells pleasant”, or “a bottle of wine costs 10 thalers” appear as sometimes true and sometimes false, depending on circumstances, is due to the fact that certain constituents of their application or use are not made explicit, leading to our disregarding that the proposition in question does not remain the same. “This”, for example, is an indexical sign with different referents depending on context.

And in the second example we assume tacitly, says Bolzano, that there is a context of time and space when we hear somebody making such a judgment (WL, §147) and the proposition therefore does not remain the same. Bolzano thus takes the pragmatic aspects of language into account.

Outside actual discourse pragmatics the proposition above - “a bottle of wine costs 10 thaler” should read: “At place X and time Y, a bottle of wine costs 10 thalers”. If it would remain true forever and everywhere in the world that “a bottle of wine costs 10 thalers” then this proposition would be analytic, according to Bolzano's definition of analyticity (WL §148). The social and objective world, as it actually happens to be, is the arbiter of analyticity and propositions can be analytic by virtue of natural laws or even by virtue of mere accidental constellations. It may obviously happen that we do not know, whether a proposition is analytic or not (see also: Kneale, W.&M. 1971, *The Development of Logic*, Oxford UP. p. 366f).

IV.

It might happen that we achieve even two different proofs – one analytic and the other synthetic - for one and the same theorem. The angle-sum theorem of a triangle provides such an example.

According to Kant, the notion of a triangle does not analytically contain the fact that the sum of its angles amounts to two right angles. So the proposition, "the sum of angles in a triangle is 180° " is synthetic, according to Kant.

Bolzano agrees. But for Bolzano all particular propositions, having the form: "This A has b", are analytic, if the general proposition "All A have b", is true, and are synthetic otherwise. Every proposition which represents just a particular example of a more general truth where the subject "belongs to a certain kind of thing" (WL §33) is analytic. We have seen that already when discussing sentences about the price of bottles of wine etc. So if we consider the angle-sum theorem of the triangle as **a special case** of the theorem:

„The angle-sum of a n-gon = $2(n-2)$ right angles“, is analytic (Otte, M. 2014, Mathematics, Logics and Philosophy, *Logique Et Analyse*, Brussels, p.83-112, p.102).

By substituting n by 3 one gets the proposition: "The angle-sum of a 3-gon = $2(3-2)$ right angles“. This proposition is analytic again, being a special case of an analytic proposition. However, mathematicians would consider this last proposition as equivalent to saying: "The angle sum in a triangle equals 2 right angles”.

Thus we have two propositions, one analytic the other synthetic and both representing the very same mathematical facts.

A grounding proof of the theorem „The angle-sum of a n-gon = $2(n-2)$ right angles“ would have to start from a triangulation of the polygon in question and would have to build a proof from the triangles that is, from the parts of the triangulation.

Bolzano considers analytical propositions as particularly useful to the pragmatics of mathematical discourse. Although they cannot serve as foundations (axioms) they are useful to enlarge our universe of discourse beyond the empirically given. Bolzano says:

„Mathematicians profit most from this generalization (to „imaginary“ representations) especially in the theory of equations. Equations are, taking the notion at face value nothing else than statements about the equivalence of two representations (Gleichgültigkeit zweier Vorstellungen). Stating that $4+5 = 11-2$, just means to say that the representation $4+5$ comprises the very same object as the representation $11-2$. Would one stop, however, at this conception, equations like $2-2 = 0$ or $1/i = -i$ and similar ones would not be admitted, although their meaning is easy to explain according to what has been said already“ (WL §108).