

# Bolzano's requirement of a correct ordering of concepts and its inheritance in modern axiomatics

The question of the right order of concepts cannot be separated from the problem of rigor in mathematics and is usually formulated with reference to Aristotle's distinction between *ordo essendi* and *ordo cognoscendi*: the search for rigor in science should include some kind of activity that could lead us from what is first for us to what is first in itself. Bolzano's remarks about correctness of definitions and proofs are based on the requirement of a right order of concepts and truths. Recent literature has devoted great attention to the objective order of propositions in proofs, explaining it by association with the theory of grounding. Yet, scarce attention has been given to the order of concepts, which is related to the form of definitions and to the distinction between simple and complex concepts. The paper will investigate whether the order of concepts should be considered as having an ontological or epistemological value, given that "a concept is called simple only if we ourselves can distinguish no more plurality in it".

Bolzano's view on the order of concepts will be reconstructed on the basis of his mathematical and logical works, in order to understand the relation between his logical and epistemological viewpoints. The ban on kind crossing as well as the use of philosophical notions (e.g. similarity) in geometrical proofs will be analyzed to verify whether a general hierarchical order of scientific concepts regulates the correct ordering of concepts in mathematics.

A comparison with Wolff's conception, and the analysis of the definition of similarity of mathematical objects, will suggest a tension, inherited from Leibniz, between the tendency to have a unique hierarchical order of all concepts and an order for each specific mathematical discipline. A further comparison with the investigations on explicit and implicit definitions developed by the Peano School will allow to establish whether, notwithstanding different syntactic formulations

of definitions, Bolzano's requirement of an order of concepts maintained some role up to Peano's axiomatics.

## References

- [1] Betti, A. (2010). *Explanation in metaphysics and Bolzano's theory of ground and consequence*. *Logique et Analyse*, 53(211):281–316.
- [2] Bolzano, B. (1837). *Wissenschaftslehre. Versuch einer ausführlichen und grösstentheils neuen Darstellung der Logik mit steter Rücksicht auf deren bisherigen Bearbeiter*. Seidel, Sulzbach.
- [3] Bolzano, B. (1981). *Von der mathematischen Lehrart*. Frommann-Holzboog, Stuttgart-BadCannstatt.
- [4] Bolzano, B. (2004). *The mathematical works of Bernard Bolzano*, ed. by S. Russ. Oxford University Press.
- [5] Centrone, S. (2016). *Early Bolzano on ground-consequence proofs*. *The Bulletin of Symbolic Logic*, 22(3).
- [6] Correia, F. and Schnieder, B. (2012). *Metaphysical grounding: Understanding the structure of reality*. Cambridge University Press.
- [7] de Jong, W. R. and Betti, A. (2010). *The classical model of science: A millennia-old model of scientific rationality*. *Synthese*, 174(2):185–203.
- [8] Johnson, D. M. (1977). *Prelude to dimension theory: The geometrical investigations of Bernard Bolzano*. *Archive for History of Exact Sciences*, 17(3):261–295.
- [9] Sebestik, J. (1992). *Logique et mathématique chez Bernard Bolzano*. Vrin, Paris.