Symposium BMMS: Did Bolzano Solve the Eighteenth Century Problem of Problematic Mathematical Entities?

During the eighteenth century, mathematics was widely regarded as the paradigmatic example of apodictic knowledge, especially by the influential philosophers Wolff and Kant. The new mathematical inventions of the seventeenth century, like infinitesimals, were considered as tools to solve problems in the natural sciences rather than as proper mathematical objects that might be the starting point of new mathematical subdisciplines. While mathematics was slowly developing itself into a field independent of its applications, the philosophy of mathematics was still dominated by Wol 's mathematical method, which is modeled after Euclid's *Elements*. At the end of the eighteenth century, several minor figures in the history of mathematics and philosophy, such as Michelsen, Langsdorf and Schultz, attempted to reintegrate the mathematical developments of the seventeenth and eighteenth century into the philosophy and epistemology of their time. An important part of their publications is devoted to the manner in which problematic mathematical entities such as infinitesimals and complex numbers should become part of mathematics.

In his early *Contributions to a better founded presentation of mathematics* of 1810, Bolzano responded to these issues by proposing a much wider conception of mathematics by rejecting the traditional definition of mathematics as the study of quantities. Notes and manuscripts of 1811 and 1812 confirm this radical departure from the tradition. Three decades later, Bolzano returned to the traditional conception of mathematics and offered a solution to the problem of problematic mathematical entities by allowing objectless ideas. While the early Bolzano responded radically to this problem by developing a quite general conception of mathematics, the later Bolzano much more carefully formulates a slightly broader conception of quantity and combines this with a quite advanced epistemology that allows objectless ideas to be meaningful under certain conditions. As a result, the later Bolzano seems to hold that a scientific (sub)discipline can have objectless ideas as its topics of study.

In this paper, I will attempt to answer the following question: why and how did Bolzano change his position and in what manner does this change relate to other developments in the philosophy of mathematics during the first decades of the nineteenth century? To this end, I will first summarize the issues concerning several problematic mathematical objects as they were discussed at the end of the eighteenth century. Subsequently, I will sketch Bolzano's early and late solutions to these problems. Most of the paper will be devoted to an investigation into Bolzano's notes and manuscripts in order to attain an understanding of why and how Bolzano's changed his solution. Finally, I will compare Bolzano's approach to responses of his contemporaries.