**BMMS: On Bolzano's early rejection of infinitesimals**

Bernard Bolzano introduced the notion of variable quantities $\omega$ in his work on the binomial theorem as an alternative to the "selbst widersprechenden Begriffe unendlich kleiner Grössen" (1816: XI). The latter, he wrote, postulated quantities that were *de facto* smaller instead of quantities that *could be* smaller than another, and it compared those quantities with any *alleged* or *conceivable* and not merely with any *given* ones (cf. 1816: V). Because of such notion and the procedures associated with it, the *Rein analytischer Beweis* (1817) of Bolzano has been traditionally considered as an "epoch-making paper on the foundations of real analysis" (Ewald, 1999: 225). That way, his definition of a continuous function as that for which "der Unterschied $f(x+\omega) - fx$ kleiner als jede gegebene Grösse gemacht werden könne, wenn man $\omega$ so klein, als man nur immer will, annehmen kann" (1817: 11-12), is usually interpreted as equivalent to the later definitions of Cauchy and Weierstrass.

In this paper we will examine Bolzano's mathematical diaries written until 1818 in order to provide a better understanding of his careful definition of quantities $\omega$, which he used in his published mathematical works from 1816-1817. As will be shown, despite the fact that Bolzano's mathematics hinted at some ground-breaking features and concerns, there is an intrinsic difference between his notion and the Weierstrassian $\varepsilon$. In particular, there seems to be enough evidence to sustain that his alternative concept was rooted in a certain distinction made between actual and potential infinite. That way, in a note dated around December 1814, he stressed that quantities $\omega$ should be understood as the assertion "dass man zu jedem schon angenommenen [Grösse] ein noch kleineres (grösseres) annehmen kann" (*BGA* 2B7/1: 79). As Bolzano's *Reine Zahlenlehre* shows, his later developments involved a "major change" (Russ & Trlifajová, 2016: 44) on the notion of infinity, as he went on to accept what he described in a note from 1818 as the "noch zweifelhaft" concept of infinitely small quantities (*BGA* 2B9/2: 126).

References

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