Recently, two 19th century constructions of the real numbers have received attention: that of Frege (Snyder and Shapiro 201X) and that of Bolzano (Russ and Trlifajová 2016). While Frege's construction is explicitly placed by Epple (2003)'s conceptual scaffolding into traditional (Frege, Hankel), formal (Hilbert, Thomae) and arithmetical (Cantor, Dedekind) constructions of the reals, it is an open question how to categorise Bolzano's.

Interpreters agree that what Bolzano calls measurable numbers are in fact the reals. If we follow Bolzano literally, numbers, including thus the measurables, are sums, i.e. a certain kind of collections. This follows from the fact that, per GL and PU, numbers are quantities (GL §1) and quantities in turn are defined in terms of sums (PU §6). Hence, the definition of measurable numbers ultimately relies on the concept of sum. In the existing literature (van Rootselaar 1964, Rusnock 2000, Russ and Trlifajová 2016), however, Bolzano's measurable numbers are discussed through set theoretic interpretations. The reasons for adopting such an interpretation are understandable, as set theory is the language of modern mathematics, and that set theoretic interpretations can thus be mathematically expedient. The downside of such choice is however that it can lead to an anachronistic understanding of the underlying philosophy of science Bolzano endorses, thus hindering efforts of placing Bolzano's measurable within a framework like Epple's.

In this talk I examine some of Bolzano's mathematical proofs and assess them in the context of Bolzano's general philosophy of science while resisting the use of set theoretic resources. Aim of this analysis is to put into starker relief the differences between Bolzano's theory of collections — sums in particular — and modern set theory, while highlighting the importance of Bolzano's conception of science for his philosophy of mathematics. By tracking the concepts Bolzano deploys in his work on the measurable numbers, we can better assess his contribution in relation to those of later mathematicians such as Frege, Dedekind and Cantor.

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