Bolzano in Prague 2017

Overview

200th Anniversary of some Mathematical Achievements of Bernard Bolzano

This was a small discussion meeting in Prague, 27th - 29th November 2017. It was informally organised by Steve Russ and Davide Crippa and was announced as part of the programme of the Czech Academy of Sciences in Prague.

The purpose of the meeting was two-fold:

(i) to celebrate the publication in 1817 of Bolzano's works, *Rein analytischer Beweis* ... and *Die Drey Probleme* ... ;

(ii) to discuss and further publicise two recent PhD theses which focus substantially on Bolzano's mathematics, one mainly from a philosophical perspective (Johan Blok) and one mainly from an historical perspective (Elías Fuentes Guillén).

There were other sessions based on papers recently published (Stefan Roski, Steve Russ & Kateřina Trlifajová) or forthcoming (Kateřina Trlifajová).

Full papers were neither invited nor presented. The meeting proceeded by discussion of existing papers or theses distributed beforehand (on the web). Support for the meeting was given through a webpage:

https://bernardbolzano.org/

and through an associated Google group and Skype. The webpage describes the schedule, and makes the source texts used available for download, among other resources.

Texts and Discussions

Theses

'Bolzano's Early Quest for A Priori Synthetic Principles : Mereological Aspects of the Analytic-Synthetic Distinction in Kant and the Early Bolzano'
(Johan Blok, PhD thesis, University of Groningen, November 2016)

'The Germanic Development of the Pre-Modern Notion of Number: From c. 1750 to Bolzano's *Rein analytischer Beweis*'
(Elías Fuentes Guillén, PhD thesis, University of Salamanca, July 2017)

Other texts discussed were as on the Schedule and Source Texts linked here and from the webpage. Two sessions of the meeting were devoted to each thesis, one session for most other texts. Details are below.

Authors were invited to summarise their presentation or session with a few sentences. Their responses, and some commentary, are given here to give the flavour of discussions.

From Johan Blok:
"During [my] first session some themes in Bolzano's early work were discussed against the German philosophical background of the eighteenth century, mainly consisting of Wolff's method of mathematics, and Kant's notion of synthetic *a priori* principles. To mention one of the topics, being aware of the dominance of Wolff's mathematical method - modeled after Euclids *Elements* - in German textbooks throughout the eigtheenth century, helps to understand and appreciate Bolzano's fierce rejection of the Euclidean treatment of geometry and his many attempts of building a new geometry by starting from simple concepts. Another important issue that came to light during the session is that in Bolzano's early work, several concepts and distinctions of his late work (WL), such as the notion of proposinition in itself (*Satz an sich*), are not yet used in a systematic manner, nor made explicit, although the underlying ideas seem to be present, although dimly, in several passages of Bolzano's early work.

The second session was devoted to a reconstruction of Bolzano's early account of arithmetic and his early theory of natural numbers. Bolzano's manuscripts written around 1810 (GA2A5), provide sufficiently detailed information to reconstruct a theory of number and arithmetic as part of general mathematics in such a way that it allows for quite a sophisticated explanation of Bolzano's claims in the *Beyträge* concerning the nature of mathematics as 'the science of forms', and the synthetic nature of arithmetic. In short, the manuscripts characterize general mathematics as 'thinking things together in thought' by means of **et** composition. They also provide a definition of natural numbers in terms of **et** composition by applying recursion, a solution to the problem of addition, and the claim that the order of addition does not matter. Taken together with Bolzano's notion of synthetic *a priori* principles of the *Beyträge*, they explain why the early Bolzano regarded arithmetic as synthetic, while also illustrating what Bolzano meant when defining mathematics as the science of forms."

A stable URL for Johan Blok's thesis at his university is
http://hdl.handle.net/11370/18ae6aa4-f9ef-49ff-9ef7-08bc66fd8915

From Elías Fuentes Guillén we give a brief selection from the Longer Summary (on the webpage):

"… the theses of my work are:

I. Major thesis: Bolzano's mathematical project until 1817 is pre- and not proto-Weierstrassian since, despite their apparent similarities, not only it is not an effort to create a "theory of $\mathbb{R}$ [...] using set-theoretic constructions [and] starting from the natural [or rational] numbers" (... arithmetization), but above all it still features traits of mathematical notions and practices that are heavily deviant from the Weierstrassian and later ones of modern real analysis.

II. Minor thesis: Bolzano's variable quantities ω are neither numbers (Weierstrassian ε) nor quantities that tend to zero in a strictly dynamic way (Cauchy α), but quantities that can become smaller than any given one and, therefore, might represent an attempt to work in 'finite' terms. "

In Section C of his thesis Fuentes gives a detailed account of why he suggests Bolzano is not anticipating a Weierstrassian-style of modern analysis including discussion of the nature of the 'variable quantities' ω and Ω used in the 1816 work.

In Section A there is valuable contextual material on the relevant political and educational situation in the later 18th century. Section B contains particularly detailed studies of three prominent German mathematicians (Segner, Kästner and Karsten) whose work either directly influenced Bolzano (Kästner), or who are good representatives of the broader mathematical context for Bolzano and his teachers.

In Section 1. of Fuentes's 'Longer Summary' he says, " … strictly speaking what is found [in RB] is not the Bolzano-Weierstrass theorem ...", and indeed in an earlier paragraph he quotes two superficially quite different results which either 'are', or are equivalent to, the said theorem. This resonates well with the remark of Paul Rusnock (on Skype during our meeting) that "a history of the B-W theorem would make a fascinating study".

Each of these substantial theses make interesting challenges to the *status quo* in assessments of Bolzano's thought. For example, Blok's thesis challenges the idea that Bolzano is mainly driven by opposition to Kant rather, Blok suggests, while also having disagreements with Kant, he was strongly positively influenced by him. We have already drawn attention to Fuentes's challenge to the 'traditional reading' of Bolzano in regard to his (alleged) anticipation of modern analysis.

For those especially interested in the historical background note that Fuentes thesis has a rather detailed bibliography on historical matters and includes the URL's for many webpages with original sources. Worth noting here too that Blok's thesis has made substantial use of the philosophical and mathematical notebooks, and other volumes of the Bolzano Gesamtausgabe Edition.

Other Contributions

The first session of the meeting, on grounding (*Abfolge*), was introduced by Stefan Roski with a Handout (as on the web) and he writes in summary:

"Bolzano boldly proclaimed that his proof of the theorem mentioned in the title of *RAB* does not merely establish the theorem's truth, but also shows why it is true by exhibiting its *grounds*. To understand the motivation behind Bolzano's proof, his critique of alternative proofs of the theorem, and perhaps also some peculiarities of Bolzano's particular proof-strategy, it is thus important to have some grip on Bolzano's conception of explanatory proofs (proofs that show *why* a theorem holds), which is rooted in his theory of grounding. Two general maxims that are often mentioned in this connection are the maxim that explanatory proofs ought to proceed from simpler to more complex truths, and the maxim that they ought to proceed from more to less general truths.

In my contribution, I attempted to show how Bolzano understands these maxims precisely, and what role they play in his critique of alternative proofs of the theorem he's concerned with in *RAB.* In this connection, I offered two interpretations of the simplicity requirement that are often not kept apart in the literature, and I pointed out that Bolzano does allow for explanatory proofs that derive more from less general truths. "

There was an unscheduled (but very welcome) arrival from Jan Sebestik, via Skype, during Stefan Roski's introduction, and he stayed until the end of discussion.

Immediately following Roski's introduction Ladislav Kvasz began discussion saying that he did not find Bolzano's proof in RB to be 'explanatory'. There followed some lively discussion

exploring the tension between the apparent subjectivity of being 'explanatory' and the objectivity expected in a 'grounding'.

There were two scheduled Skype sessions: a shorter one on Monday with Elías Fuentes Guillén to make sure our system worked, and a longer one, recorded, on Tuesday. Since the latter was four hours long (apparently with no break!) we cannot do justice to it here. It was enjoyable, with Elias in Mexico, Paul Rusnock in Canada and Jan Sebestik in Paris. Maybe on the website some summary will be posted in the next week or two.

The Wednesday morning saw the final two presentations, one from Steve Russ, and one from Kateřina Trlifajová; these were more mathematical in nature. The first was a joint paper about Bolzano's work on measurable numbers (from the 1830's, see Source texts), where the emphasis of the presentation (not so much of the paper) was on the difficulties of presenting work that was unfinished, and remained unpublished for well over a century. The work clearly shows remarkable individual insight, and most of the technical work needed, to reach two major conceptual milestones: identifying the algebraic field structure and the construction of real numbers. The latter step was taken in many ways many decades later with no awareness of Bolzano's achievements.

The presentation from Kateřina Trlifajová was the closest to a conventional 'lecture' which was appropriate as it was a substantial and ambitious paper – now a forthcoming publication. Few of us could have read it in detail, so it was welcome and useful to hear the main points outlined to us. The idea is to explore whether the Euclidean principle of 'the whole is greater than the part' (which is sacrificed in Cantor's theory of the infinite) can in fact be preserved consistently in a theory of infinite objects. The paper aims to show that this can indeed be done, and even in a variety of different ways. It deserves much more discussion than we could give to it at the time.

Outcomes

Several participants expressed interest before the meeting in having tangible outcomes from our discussions. This report is an outcome for the IBBG Newsletter. It is possible that a shorter version will be prepared for inclusion in issues of *Historia Mathematica* and *Philosophia Mathematica*. The website and the Google group are also outcomes which both have a potential future for interaction and contribution. We hope to prepare some research issues in the form of questions, for the website, and these could be pursued by any people interested through the Google group. There were many issues that we did not have a chance to discuss properly at the meeting. Any other suggestions for outcomes, or proposals for further collaborations, are welcome.

Steve Russ and Davide Crippa (Co-organisers of BiP17)

27th December 2017 (updated 10th January 2018)